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Semiempirical Model for Satellite Energy-Accommodation Coefficients

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The energy-accommodation coefficient is an important parameter affecting satellite drag and orbit predictions. Previous estimates of this coefficient have been based on interpolation from values tabulated at several altitudes and solar conditions. In an effort to improve drag coefficient accuracy and to compute values of the accommodation coefficient that respond to the real variability of the atmosphere, a first-principles approach is desired. The present work combines the theory that gas–surface interactions in low Earth orbit are driven by adsorption of atomic oxygen, with observations of satellite accommodation collected during solar cycle 22. The result is a semiempirical model based on Langmuir's adsorption isotherm, which agrees with the data to within 3%. This model can be used to improve drag predictions during a wide range of space weather conditions, as well as to improve the accuracy for atmospheric densities derived from satellite drag.

Nomenclature

A = surface area, m^2
A_p = planetary geomagnetic index
C_D = drag coefficient
E = kinetic energy, J
F = force, N
$F10.7$ = solar flux of 10.7 cm wavelength radiation, solar flux units
K = model-fitting parameter, m^3/K
k_b = Boltzmann constant, m^2 kg s^{-2} K^{-1}
m = molecular mass, kg
N = total number of atmospheric species under consideration
n = number density, m^{-3}
P = partial pressure of atomic oxygen divided by the Boltzmann constant
$\mathbf{r}$ = location of elemental surface area in the body reference frame, m
s = speed ratio
T = temperature, K
v = velocity magnitude, m/s
α = global energy-accommodation coefficient
α_s = local energy-accommodation coefficient
β = reciprocal of the most probable speed, s/m

Subscripts

i = incoming (freestream) molecules
j = atmospheric species index
k = kinetic
O = atomic oxygen
r = reflected molecules
w = wall conditions

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Introduction

SATELLITE drag coefficients are the primary source of uncertainty in atmospheric drag prediction, as well as in the deduction of atmospheric densities from satellite drag. The drag coefficient is heavily dependent on the interaction of the atmospheric gas with the satellite surface. The energy-accommodation coefficient α is a key factor in this interaction and can be thought of as the average fraction of energy lost by molecules impinging on the surface. This parameter has previously been determined empirically for satellites orbiting in the Earth's thermosphere, and tabulated values exist as a function of altitude for solar-maximum and solar-minimum conditions [1–3]. At the time of writing, the accommodation data represent the state of the art in computing drag coefficients that respond correctly to the changing environment at a range of altitudes in the upper atmosphere. When applying the existing information, however, an altitude fit is performed to a set of values that may not correspond to the solar conditions in question. The purpose of this paper is to describe the first energy-accommodation model consistent with satellite-accommodation data that can be applied at all levels of solar activity while taking into account some of the basic physics of gas–surface interactions. Rather than compiling a series of ad hoc fits along the altitude component at a variety of atmospheric conditions, the formulation described herein has a theoretical underpinning that makes it generally applicable while fully leveraging the available data. It enables the computation of α between all solar-maximum and solar-minimum conditions, as well as extrapolation to extreme space weather behavior, such as the deep minimum of solar cycle 23.

Early laboratory measurements of gas–surface accommodation have been fraught with uncertainties caused mainly by the conditions and cleanliness of the surface [4]. Furthermore, other researchers have noted that conditions on the satellite surface are not represented adequately in the laboratory [5]. One of the reasons for this discrepancy is that atomic oxygen in the thermosphere adsorbs onto satellite surfaces. Instead of interacting with a metallic lattice or the glass surface of a solar cell, incoming molecules collide with the lighter adsorbate species, resulting in a more inelastic collision. Atomic oxygen adsorption has been confirmed by observation from pressure gages [6] and is the primary reason for the altitude dependence of the drag coefficient [3,7]. The pressure gauge measurements suggest that time scales for adsorption and desorption and, therefore, changes in energy accommodation could be on the order of seconds. This response time has not been measured in the context of drag coefficient changes, so it is not possible to verify its applicability at the present time. The determination of characteristic time scales for changes in satellite accommodation is beyond the scope of this paper and left as a subject of future work.

The accommodation coefficient is formally defined as [8]

$$\alpha = \frac{T_{k,i} - T_{k,r}}{T_{k,i} - T_w} \tag{1}$$

where $T_{k,i}$ is the kinetic temperature carried to the surface by an incoming molecule, $T_{k,r}$ is the kinetic temperature of the reflected molecule, and T_w is the kinetic temperature the molecule would have if it was reemitted at the temperature of the surface [9]. Kinetic temperature can be written as

$$T_{k,i} = \frac{m v_i^2}{3 k_b} \tag{2}$$

The reflected kinetic temperature at the surface is

$$T_{k,r} = \frac{m}{3 k_b} v_i^2 (1 - \alpha) + \alpha T_w \tag{3}$$

Equation (3) holds exactly for monatomic species, but the expected error in applying it to diatomic molecules is less than 1%. Finally, we can insert the reflection temperature into the free-molecular flow equation for drag on a sphere [9]:

$$C_{D,\mathrm{sphere}} = \frac{2s^2 + 1}{\sqrt{\pi} s^3} \exp(-s^2) + \frac{4s^4 + 4s^2 - 1}{2s^4} \mathrm{erf}(s) + \frac{2\sqrt{\pi}}{3s} \sqrt{T_{k,r}/T_i} \tag{4}$$

where the speed ratio s is

$$s = v_i \beta \tag{5}$$

and $\beta = \sqrt{m/(2 k_b T_i)}$. The erf() denotes the error function, defined as

$$\mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\,\mathrm{d}t \tag{6}$$

The physical interpretation of β is a reciprocal of the most probable thermal speed. The temperature T_i in the definition of β and in Eq. (4) refers to the thermal temperature of the ambient gas, irrespective of its bulk motion, and is not to be confused with the incident kinetic temperature in Eq. (2). Computing the drag coefficient, using Eq. (4), with an expected low-Earth-orbit accommodation range of 1.00 to 0.70, an atmospheric temperature of $T_i = 1100$ K, a surface temperature of $T_w = 300$ K, a relative atmospheric speed of $v_i = 7,800$ m/s, and molecular mass of 18 atomic mass units results in a drag coefficient range of 2.12–2.57. This represents a 20% variability due to changes in accommodation alone.

The most direct measurements of satellite energy accommodation come from independent measurements of satellite drag, such as orbital decay and spin decay. By measuring drag and drag torque simultaneously, satellites in the shape of paddle wheels have been used to deduce both density and the accommodation coefficient [10,11]. Energy accommodation may also be estimated by comparing observed drag with the drag predicted by a calibrated atmospheric model. Adsorption theory states that the fraction of adsorbed molecules can be a function of adsorbate partial pressure and surface temperature, both of which can be estimated in orbit [12]. Therefore, if a suitable relationship between the source of adsorption and α measurements can be found, it can then be used as a basis for modeling energy accommodation. In the subsequent sections, we describe how the available accommodation measurements can be used to construct such a model.

Model Assumptions

The first assumption underlying this work is that the energy-accommodation coefficient in Earth's lower orbits (below 500 km altitude) is driven primarily by the amount of atomic oxygen adsorbed on the surface. Therefore, adsorbed molecules shield the incoming flow from surface properties, such as treatment and composition [13]. Another important assumption is that molecules are reflected from the surface in a diffuse, or cosine, distribution with a kinetic temperature $T_{k,r}$ determined by Eq. (3). According to previous work on satellite drag coefficients, this assumption is valid for satellites flying below 500 km altitude [5].

So far, we have discussed the accommodation coefficient as a single quantity to be applied to the entire satellite geometry. This is the assumption inherent in Eqs. (3) and (4), which develop the drag coefficient of a sphere as a function of α. It is possible that, due to variations in flow shadowing and angle of attack, the value of α changes over the surface of a satellite. For example, consider a flat plate oriented normal to its velocity together with one that is oriented parallel to its velocity flying in the upper atmosphere. The flux onto the two surfaces will depend on the temperature of the atmosphere, the angle of attack, and the magnitude of the velocity. Under an expected range of satellite conditions ($7,000$ m/s $< v_i <$ $10,000$ m/s and 500 K $< T_i <$ 1500 K), the surface flying normal to the flow will experience a higher flux and adsorption and would, therefore, have a higher accommodation coefficient according to the assumptions stated thus far. Likewise, on the surface of a satellite, elements parallel to the flow would have a lower accommodation than those facing the flow. However, because the drag coefficient of a plate at a low angle of attack is lower than that of a plate flying normal to the flow, the averaged or effective drag coefficient would be closer to that of the surface elements facing into the flow. The result is that the effective accommodation coefficient is dominated by the accommodation of the surface elements with the highest contributions to drag. This averaging is also applied over the various species j of the atmosphere and incoming kinetic energies E. Orbital measurements of α are, therefore, a result of a weighted average of the accommodation coefficients for each species and energy in the atmosphere. Equation (7) formally expresses the idea of this effective parameter as the integration of surface-element drag coefficients over the surface of a satellite geometry:

$$\frac{\int C_D(\alpha, \mathbf{r})\,\mathrm{d}A}{A} = \frac{\sum_{j=1}^{N} (\{\iint C_D[\alpha_{s,j}(\mathbf{r}, E), E, \mathbf{r}]\,\mathrm{d}A\,\mathrm{d}E\} n_j m_j)}{A \sum_{j=1}^{N} n_j m_j \int \mathrm{d}E} \tag{7}$$

Here, $\mathbf{r}$ refers to the position of each surface element with respect to the flow, and n_j and m_j are the atmospheric species number densities and masses, respectively. The inner integral on the right side is performed over the surface of the satellite and the outer integral over the applicable range of energies for species j. While the effective parameter α is a global constant for any particular geometry, $\alpha_{s,j}$ is a surface-, energy-, and species-dependent property and would coincide closely with laboratory measurements if they could reproduce the adsorption environment in Earth's thermosphere. The effective parameter α corresponds to a constant value measured in orbit for a particular satellite and for specific flow conditions. From here, we will refer to the effective accommodation as simply accommodation or α, and we will take this to signify a parameter that is constant over the surface of the satellite.

Observations

Accommodation coefficients for solar minimum and maximum are shown in Fig. 1 [1–3,10,14,15]. The empirical values indicated by unbarred data points (Pardini et al. [2], Bowman and Moe [1], and Moe et al. [3]) have been computed by comparing satellite drag measurements with adjusted atmospheric models to deduce observed drag coefficient values. The observed drag coefficients at several altitudes were then modeled physically to invert the appropriate accommodation coefficients reproduced in Fig. 1 [2]. Values obtained in this way will be referred to as fitted accommodation coefficients. It is important to understand that the fitted coefficients used in this analysis depend directly on the atmospheric model and indirectly on a priori assumptions about accommodation as a function of altitude. The reason for this dependence is that the biases in the density model are themselves estimated based on satellite drag observations [1]. Model bias is estimated by comparing fitted drag coefficients with theoretical values that are based on existing

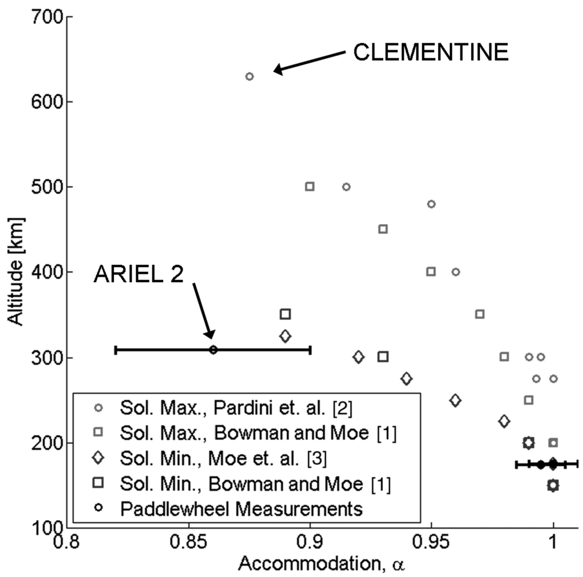

Fig. 1 Estimated energy-accommodation coefficients.

knowledge of accommodation coefficients. Therefore, the resulting energy-accommodation values are not absolute and independent but, instead, relative to the atmospheric model and the a priori assumptions of accommodation altitude profiles. Other methods of accommodation measurement include paddle-wheel satellites (black circles), which measured spin decay and drag or drag and lift simultaneously. The horizontal bars in Fig. 1 indicate the reported uncertainty in the paddle-wheel measurements. The uncertainty in the fitted accommodation results are estimated at approximately $\pm 3\%$. Two data points, one fitted and one measured via paddle wheel, are indicated in Fig. 1. These points are outliers, which are located away from the concentration of data. The fitted outlier corresponds to the European Clementine satellite (not the interplanetary mission) that had a complicated shape with deployable solar panels [2]. The paddle-wheel outlier is the Ariel 2 satellite. The Ariel 2 measurement is 7% lower than the fitted drag coefficients at a similar altitude. The discrepancy could be due to measurement uncertainties or because Ariel 2 flew at the least active time during the solar cycle. It is also possible that gas–surface interactions are a function of incident molecule speed (kinetic energy). In this case, fitted accommodation coefficients of objects in near-circular orbits would be expected to differ from the Ariel 2 observation, which corresponds to a 400 m/s increase in incident speed.

The drag study by Bowman and Moe [1] analyzed a number of spherical objects during the 1989–1995 time frame covering both solar minimum and maximum. These spherical objects (listed in Table 1) were used by Bowman and Moe [1], along with the high-accuracy satellite drag model [16], to infer the accommodation coefficients at several altitudes during both solar minimum and solar maximum. Their results are shown in Table 2. The satellites flying before 1990 correspond to solar maximum, while those flying after 1990 correspond to the solar minimum of solar cycle 22. Based on orbit altitude and the binary parameter of solar activity (sunspot maximum or minimum), the accommodation coefficients for these objects can be determined from Table 2. These tabulated values are used in the work described here to find the dependence of accommodation on modeled atomic oxygen density.

Computational Method

To account for the temporal variations in solar conditions, a functional relationship between time-specific atmospheric conditions and the accommodation coefficient is desired. This will allow the computation of accommodation coefficients for any arbitrary time. Since the physical explanation for accommodation is based on atomic oxygen adsorption, a modeled parameter based on atomic oxygen partial pressure would be ideal. The product of the density of atomic oxygen and atmospheric temperature ($n_O \cdot T_i$) was chosen as the input parameter. Both atomic oxygen number density n_O and atmospheric temperature T_i are modeled, using NRLMSISE-00 [17] with solar activity inputs (A_p and $F10.7$) taken from measurement archives. The measurement and modeling of atomic oxygen number densities has presented many challenges, such as recombination of atomic oxygen inside of mass spectrometers and uncertainties in the ionization cross section. Since NRLMSISE-00 depends on mass spectrometer data for this information, our model cannot be fixed to an absolute input scale. Instead, we will rely on a relative estimation of the atomic oxygen, assuming that the relative errors are smaller than the absolute accuracy within the atmospheric model.

First, the orbits of seven spherical satellites used by Bowman and Moe [1] were propagated at various altitudes and times during 1989–1995 to find the average atmospheric conditions along their trajectories. The spherical tracking objects used are listed in Table 1. The accommodation coefficients that were ascribed to those spheres (via interpolation in Table 2) were then matched with average values of $n_O \cdot T_i$, as computed by the NRLMSISE-00 model in our orbital simulations. This was done for each satellite at three to seven different altitudes, resulting in 38 data points spanning a range of altitudes and solar conditions. Finally, a least-squares fit was used to match an analytical model to the resulting data set. The chosen analytical model is the Langmuir isotherm, which describes the fraction of monolayer surface adsorption by a particular species as a function of the partial pressure of that species [12] at a certain constant surface temperature. This model was selected because of its simplicity and because it has been successfully used to reconcile spaceborne pressure-gauge observations of atomic oxygen [6]. Equation (8) is the definition of the Langmuir isotherm in terms of a scaled partial pressure component (partial pressure divided by Boltzmann's constant) P and a single constant K, which is also used as the fitting parameter. The value of P was set equal to $n_O \cdot T_i$. We have assumed a linear relationship between surface coverage of atomic oxygen and energy accommodation. In other words, the model describes a macroscopic accommodation coefficient that is an average of unaccommodated collisions between incoming molecules and the spacecraft material and completely accommodated collisions between the incoming molecules and the adsorbate layer. This assumption allows us to write accommodation directly in the form of the Langmuir formula:

Table 1 Spherical tracking objects used to construct the accommodation coefficient model

Type	Ref. no.	Obs. year	Alt. range, km	Inc.
Taifun-Yug	11796	1989	206–277	82.9
Taifun-Yug	13750	1989	232–377	65.8
Taifun-Yug	15446	1990	232–355	65.8
Calsphere	04958	1989	355–501	88.3
Taifun-Yug	21190	1995	205–302	65.8
ODERACS[a]	22990	1994	230–327	56.9
ODERACS[a]	22994	1994–1995	208–326	56.9

[a]ODERACS denotes Orbital Debris Radar Calibration Sphere.

Table 2 Accommodation coefficients computed by Bowman and Moe [1]

Altitude, km	α solar max.	α solar min.
150	1.00	1.00
200	1.00	0.99
250	0.99	0.97
300	0.98	0.93
350	0.97	0.89
400	0.95	——
450	0.93	——
450	0.90	——

$$\alpha = \frac{K \cdot P}{1 + K \cdot P} \tag{8}$$

The computational method is summarized in Fig. 2. In essence, we have presented the available accommodation measurements as a function of simulated atomic oxygen partial pressure (scaled by k_b), then we fitted an adsorption-derived function to the data, resulting in a semiempirical model of energy accommodation.

Results

The semiempirical model, along with data from spherical satellites during both solar maximum and minimum, are shown in Fig. 3 and Tables 3 and 4. The tabulated data include specific spherical-object reference numbers (North American Aerospace Defense Command references), the times of simulation, the average altitude during that time, as well as the value of $n_O \cdot T_i$ and the accommodation coefficient. The black solid curve in Fig. 3 represents an analytical fit of the results to the Langmuir isotherm. The average error between the accommodation coefficient determined through the analysis of smooth spheres and the Langmuir isotherm fit with $K = 7.50 \times 10^{-017}$ is -0.9%, with a standard deviation of $\pm 1.7\%$. This agreement gives a physical underpinning to the extrapolation scheme and validates the assumption of a straightforward relationship between energy accommodation and the fraction of atomic oxygen adsorbed at the surface. Note the deviations of the accommodation curve from the Langmuir isotherm, especially at the knee of the curve, where the model is systematically higher than the data. This may be indicative of secondary adsorption effects, such as multilayer formation (islands) [4,12] or recombination of atomic oxygen. Other explanations include relative errors in the modeled number density of atomic oxygen in NRLMSISE-00 and uncertainties in the original tabulation of accommodation coefficients. The improvement of the model in this region is left as a subject of future work. Note that the fitting parameter K depends on the method of n_O and T_i estimation. The value of 7.50×10^{-017} obtained for K in this paper is a result of using the NRLMSISE-00 neutral density model, and the application of this value should only be performed with NRLMSISE-00. The use of a different atmospheric model for the estimation of n_O and T_i would require a recomputation of K.

To compare the model with tabulated data, we compute accommodation coefficients along circular orbits of a 30° inclination at a range of altitudes for both solar minimum [$A_p = 5$ and $F10.7 = 75$ solar flux units (sfu)] and solar maximum ($A_p = 10$ and $F10.7 = 225$ sfu). Figure 4 demonstrates the agreement with accommodation measurements (discrete points). The model successfully predicts the gradual decrease in accommodation as the altitude changes from 150 to 200 km and the more rapid drop, which occurs above 300–400 km altitude. Relative differences between

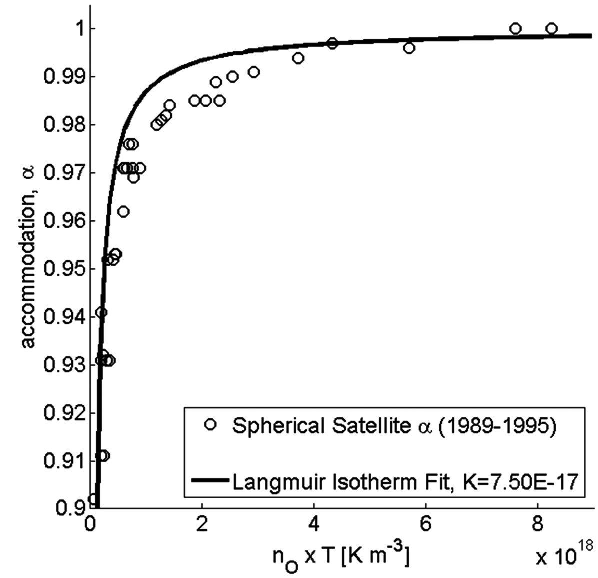

Fig. 3 Accommodation coefficients as a function of $n_O \cdot T_i$.

solar minimum and maximum are also reproduced. Most measurements are within 50 km of the model curve, and the best agreement occurs below a 500 km altitude. A notable outlier is the solar-maximum measurement at 630 km, corresponding to the Clementine satellite [2]. One explanation for this is that the adsorption model ceases to accurately represent the physics of gas–surface interactions above 500 km. It is possible, for example, that when a significant fraction of the surface is exposed to the freestream at higher altitudes, the accommodation coefficient will be determined to a larger extent by the particular material of the satellite surface. This material, or substrate, accommodation coefficient can take on values ranging from 0.0 to 0.9, depending on the ratio of molecular masses of incoming and surface molecules and the incident angle [5]. In contrast, the present model implicitly assumes a substrate accommodation coefficient of 0.0 for all cases. The likely consequence of this is that our model underestimates the accommodation coefficient at higher altitudes, such as those of the Clementine satellite. Also, Clementine had a complex shape, which tends to introduce uncertainty into drag coefficient computations. While this may account for some of the discrepancy, it is important to keep in mind that, because of the noncommutative nature of Eq. (7), values derived using spherical satellites may not exactly match the values measured using other shapes. Furthermore, the modeled curves in Fig. 4 correspond to average atmospheric conditions and may not adequately represent the orbital environment for specific observation

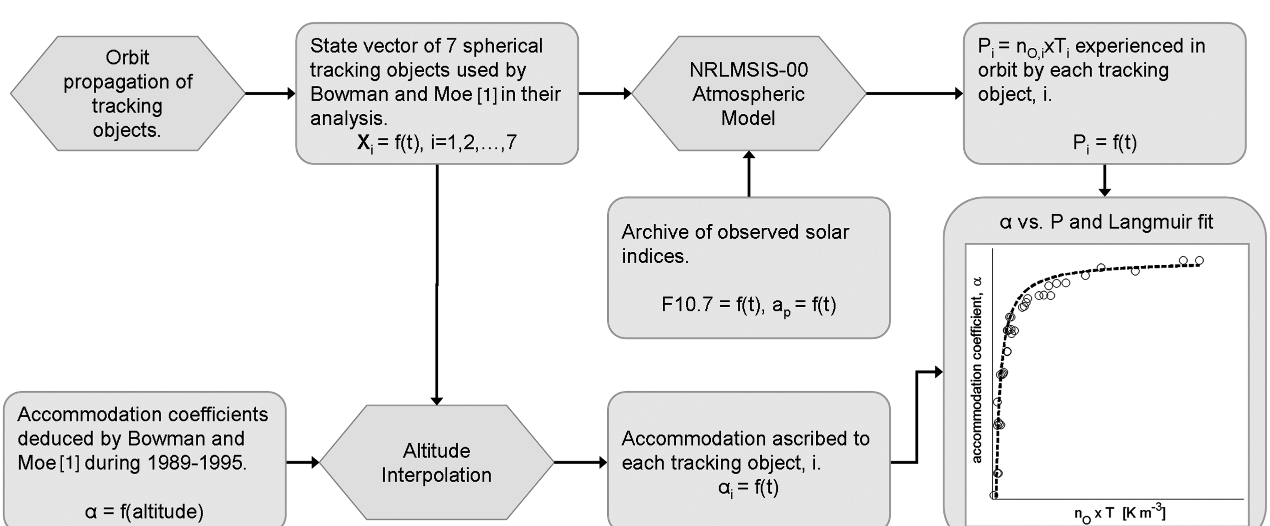

Fig. 2 Flow diagram summarizing the computation procedure for accommodation coefficients.

Table 3 Solar-maximum values of $n_O \cdot T_i$ and energy-accommodation coefficient

Obj.	Year	doy_o	doy_f	Alt.	$n_O \cdot T_i$	α
11796	89	22	24	272	1.86×10^{15}	0.985
11796	89	68	78	225	5.70×10^{15}	0.996
11796	89	165	171	198	7.60×10^{15}	1.000
13750	89	68	77	372	6.02×10^{14}	0.962
13750	89	151	155	347	5.93×10^{14}	0.971
13750	89	199	203	322	6.86×10^{14}	0.976
13750	89	232	236	296	1.27×10^{15}	0.981
13750	89	253	255	272	2.32×10^{15}	0.985
13750	89	263	266	234	3.71×10^{15}	0.994
13750	89	270	273	192	8.24×10^{15}	1.000
15446	89--90	354	3	347	6.67×10^{14}	0.971
15446	90	31	35	322	7.52×10^{14}	0.976
15446	90	58	63	298	1.19×10^{15}	0.980
15446	90	81	84	271	2.06×10^{15}	0.985
15446	90	91	93	249	2.55×10^{15}	0.990
15446	90	99	102	218	4.32×10^{15}	0.997
4958	89	174	176	498	6.69×10^{13}	0.902
4958	89	227	229	448	1.96×10^{14}	0.931
4958	89	240	242	423	1.95×10^{14}	0.941
4958	89	248	250	395	4.07×10^{14}	0.952
4958	89	253	254	372	5.88×10^{14}	0.962
4958	89	257	258	346	7.56×10^{14}	0.971

Table 4 Solar-minimum values of $n_O \cdot T_i$ and energy-accommodation coefficient

Obj.	Year	doy_o	doy_f	Alt.	$n_O \cdot T_i$	α
21190	94	339	342	298	2.89×10^{14}	0.931
21190	95	39	41	273	4.61×10^{14}	0.953
21190	95	69	71	249	8.84×10^{14}	0.971
21190	95	80	82	226	1.41×10^{15}	0.982
21190	95	88	89	198	2.92×10^{15}	0.991
22990	94	70	73	323	2.02×10^{14}	0.911
22990	94	145	148	298	2.35×10^{14}	0.932
22990	94	222	225	274	3.17×10^{14}	0.952
22990	94	245	248	249	6.12×10^{14}	0.971
22990	94	262	265	221	1.42×10^{15}	0.984
22994	94	106	108	323	2.41×10^{14}	0.911
22994	94	288	291	299	3.44×10^{14}	0.931
22994	94	352	355	273	4.39×10^{14}	0.953
22994	95	33	36	251	7.69×10^{14}	0.969
22994	95	48	50	225	1.36×10^{15}	0.982
22994	95	56	57	202	2.24×10^{15}	0.989

objects, such as Clementine. As mentioned earlier, the Ariel 2 paddle-wheel accommodation measurement at a 308 km altitude is lower than both the general trend of the solar-minimum data set and the model prediction. One possibility for this could be an error in the reported atmospheric model bias at the Ariel 2 altitude. Furthermore, the orbiting objects used to construct our model had near-zero eccentricities, with differences in perigee and apogee heights of no more than 50 km. Meanwhile, Ariel 2 was inserted into an orbit with an eccentricity of approximately 0.07 and a difference between perigee and apogee heights of roughly 1000 km. The eccentric orbit accounts for approximately a 400 m/s increase in incident kinetic energy at perigee, which could change the adsorption kinetics and lower the overall accommodation coefficient. Finally, the paddle-wheel analysis of Ariel 2 included 17 days [10] of data, while the analysis of the spherical tracking objects via fitted ballistic coefficients usually includes several months of data. This shorter time span, coupled with the atmospheric sampling experienced in a more eccentric orbit (where perigee and apogee altitude differ by more than several scale heights), could have resulted in differences in the averaged atmospheric conditions of the Ariel 2 measurement relative to the fitted ballistic coefficient analyses used in the present model. The accommodation coefficient of Ariel 2 may, therefore, be different from the accommodation coefficients of other objects at similar altitudes due to differences in average temperature and atomic oxygen partial pressure in orbit.

Equation (9) describes the implementation of the accommodation model. Both n_O and T_i are retrieved from NRLMSISE-00 for a particular set of locations, times, and solar conditions:

$$\alpha = \frac{7.50 \times 10^{-17} \cdot n_O \cdot T_i}{1.00 + 7.50 \times 10^{-17} \cdot n_O \cdot T_i} \tag{9}$$

Physically, this equation holds for all values of $n_O \cdot T_i$ and for satellite surface temperatures of approximately 300 K. Since a variation of ± 100 K in the surface temperature results in a change in drag coefficient of approximately $\pm 1.0\%$, the uncertainty due to surface temperature in the drag coefficient equation is small. It is more difficult to assert how the isotherm might change due to an increase or decrease in the surface temperature. One could theorize that, due to the chemical nature of the atomic oxygen surface bond, the bond energy will exceed the thermal energy in the surface lattice, thereby making adsorption insensitive to the changes in satellite temperatures over the free-molecular regime. At the present time, however, insufficient data on the variation of satellite-accommodation coefficients with surface temperature exist to adequately answer this question.

In terms of data validation, there are no orbital accommodation data that exist for orbits of low eccentricity at values of α lower than 0.85. An estimate for the verified range of this model is, therefore, between 0.85 and 1.00 in accommodation and 150 to 500 km in altitude. Another restriction is the eccentricity of the satellite orbit. The measured accommodation coefficient of Explorer 6 (eccentricity of 0.76) was 0.65 [5]. In contrast, the accommodation of Ariel 2 (eccentricity of 0.07), with a comparable perigee altitude to that of Explorer 6, was measured to be 0.86. One possibility for this difference is that atomic oxygen impinging at high kinetic energies does not adsorb as readily as atomic oxygen approaching the surface near or below the energy of adsorption. Another reason may be that, at high eccentricities, the time spent within the Earth's lower atmosphere is insufficient for significant adsorption to occur. However, the value measured for satellite S3-1 [14], with an eccentricity of 0.22, was consistent with another measurement at similar altitude and lower eccentricity, indicating that time spent in the lower atmosphere is less important than the energy with which molecules strike the surface and the partial pressure of atomic oxygen. Because of the Explorer 6 discrepancy, the application of Eq. (9) should be restricted to orbits with eccentricities less than or equal to 0.07. Analysis of the data in Fig. 3 indicates that the model tends to overestimate the accommodation coefficient for most values of $n_O \cdot T_i$ with a maximum deviation between data and the model of -0.04 and $+0.07$. An estimate of the maximum model error is, therefore, -0.04 and $+0.07$ around the values resulting from Eq. (9), with the accommodation coefficient restricted to values between 0.00 and 1.00. The mean error between the data and the model is -0.01, with a standard deviation of ± 0.02.

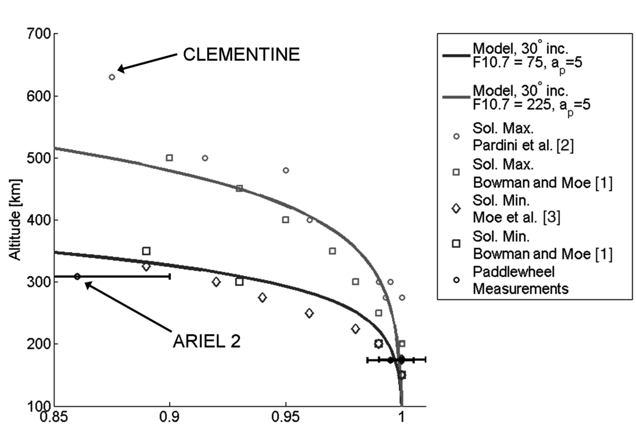

Fig. 4 Estimated energy-accommodation coefficients with the semi-empirical model.

Unlike the fitted accommodation coefficients, paddle-wheel measurements (black circles in Fig. 4) are independent of atmospheric models, which makes them ideal for confirming the accuracy of the accommodation model. Unfortunately, only five independent measurements of accommodation are available. Furthermore, the incident kinetic energies corresponding to these measurements vary, along with the partial pressures of atomic oxygen. While accommodation coefficients may depend on both of these variables, the present model presents accommodation coefficients as a function of partial pressure at a constant kinetic energy corresponding to low-eccentricity orbits. This lack of control in the kinetic energy in the paddle-wheel data set probably prevents direct comparison of model and paddle-wheel measurements at this time. It is encouraging, however, that paddle-wheel observations below 200 km match the model prediction within 1%. Because of the accommodation coefficient assumptions underlying the determination of the atmospheric model bias, the fitted accommodation coefficients reflect only relative variations in accommodation. Therefore, our model represents only a relative variation in the accommodation coefficient above 200 km.

When using the accommodation model to compute physical drag coefficients [for example, by using Eq. (4)] and applying the results to predict satellite drag forces, one must also correct the atmospheric model being used for altitude-dependent density biases. Density biases for the J70 and JB2006 models are presented by Pardini et al. [2], as well as Bowman and Moe [1]. Some biases in the NRLMSIS-00 model are presented by Picone et al. [17].

Conclusions

The energy-accommodation model presented in this paper is able to reproduce tabulated accommodation coefficients with an average error of -0.01 ± 0.02, relative to fitted accommodation coefficients. The primary assumption made in constructing the model was that the fraction of surface covered by adsorbate is directly proportional to the accommodation coefficient. This implies that the accommodation coefficient on the adsorbate is nearly unity, while the accommodation on the substrate is nearly zero. Based on the quality of the agreement between model and data, this assumption does not appreciably affect the ability to predict accommodation coefficients for Earth satellites. One straightforward improvement to the model is to take into account a nonzero accommodation coefficient on the substrate. Presently, the accommodation model is only validated for spacecraft in low-eccentricity orbits (eccentricity less than 0.07) and altitudes below 500 km. Furthermore, it is recommended that accommodation values below 0.85 be disregarded in modeling satellite drag. This is because there are presently no accommodation measurements available below this threshold for circular orbits, and the model cannot be validated until such measurements are made. The semiempirical model presented here is based on fitted accommodation coefficients that depend on atmospheric model and model bias assumptions. The fitted values are also subject to errors inherent in the determination of the satellite drag force. Because of this, the model results are not absolute values but relative to the accommodation profiles used to derive atmospheric model bias. While there is much work to be done in future accuracy improvements and expansion of the model to higher altitudes and eccentricities, it can presently be used to improve satellite drag predictions, given real-time space weather inputs. By allowing for the computation of drag coefficients in the free-molecular regime, which account simultaneously for observed changes in solar activity and altitude, the model enables researchers and engineers to forgo the ad hoc assumption of a constant accommodation coefficient over the span of satellite lifetimes. This, in turn, improves accuracy in both the prediction of drag forces as well as the determination of atmospheric densities from satellite drag.

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